Physics Graduate School Qualifying Examination

January 8, 1998

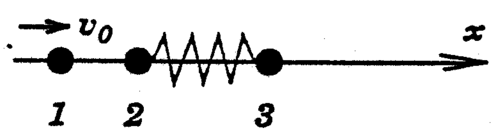
Part I

Instructions: Work all problems. This is a closed book examination. Calculators may not be used. Start each problem on a new pack of yellow paper and use only one side of each sheet. All problems carry the same weight. Write your student number on the upper right-hand corner of each answer sheet. All sheets which you receive should be handed in, even if blank.

I-1. Consider the elastic, one-dimensional collision at time $t=0$ of particle 1 (initial velocity v_0) with a two-particle system consisting of particle 2 (initally at rest at the origin) and particle 3 (initally at rest at $x=L$). Particles 2 and 3 are connected by a massless spring with elastic constant k and unstretched length L. All particles have the same mass m.

a Determine the position and velocity of each mass immediately after the collision.

b Determine the position of each mass for all $t>0$.

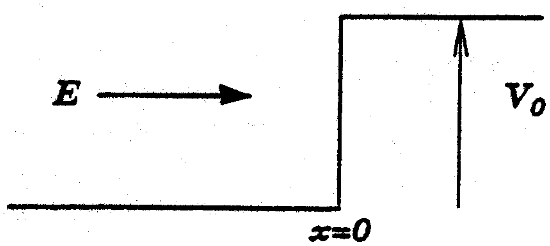


I-2. A point particle moves in a circular orbit (radius R) in a central radial force field $F(r)$. What condition must $F(r)$ satisfy if $r(t)$ is to oscillate with simple harmonic motion about R when the particle is displaced slightly from its original orbit?

I-3. An initially uncharged conductor is charged via repeated contacts with a second conductor. On the first contact, conductor one acquires a charge q from conductor two. Before the initial and all subsequent contacts, conductor two holds the same charge Q; charge is added between contacts as needed. Determine the final charge on conductor one.

I-4. An uncharged dielectric sphere with dielectric constant k and radius a is placed in a uniform electric field E_0 which exists in vacuum. Calculate the electric field inside the sphere.

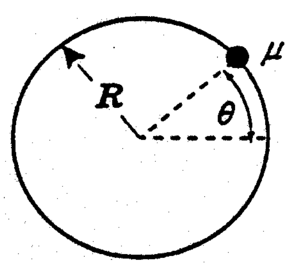
I-5. A particle of energy E is incident from the left and encounters a potential step of height V_0, where $E<V_0$.



In terms of the incoming amplitude A, determine the complete wave function for all values of x.

I-6. Consider the quantum-mechanical motion of a small bead of mass μ which slides freely along a circular wire of radius R. The Schroedinger equation for this system is

$$-\frac{\hbar^2}{2\mu R^2}\frac{d^2\psi(\theta)}{d\theta^2} = E\psi(\theta)$$



a What are the allowed energy values?

b For each energy value, what is the corresponding normalized $\psi(\theta)$?

c What are the allowed values for the angular momentum?

I-7. A point charge Q_2 (and mass M) is allowed to move in the vertical direction directly above a fixed point charge Q_1. Both charges have the same sign. A uniform downward gravitational field g exists everywhere.

a Derive an expression for the equilibrium altitude of Q_2.

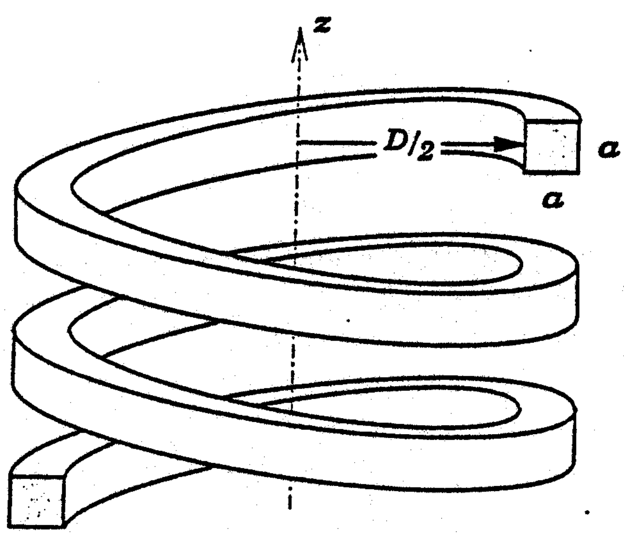
b Derive the frequency for small oscillations about the equilibrium position.

I-8. A long cylindrical solenoid (inside diameter D and length L) of N turns is wound with wire which has a square cross section of edge length a. A constant current I passes through the solenoid. Assume that $L>>D>>a$.

a Calculate the outward pressure exerted on the solenoid wires.

b Calculate the energy stored in the solenoid. (Neglect the contribution from the space within the wires.)

c The solenoid coil may stretch along its length. It behaves like a spring with force constant k and equilibrium length L_0 (when $I=0$). Calculate the change in length of the solenoid when $I\neq 0$. Assume $\Delta L<<L_0$.



Physics Graduate School Qualifying Examination

January 9, 1998

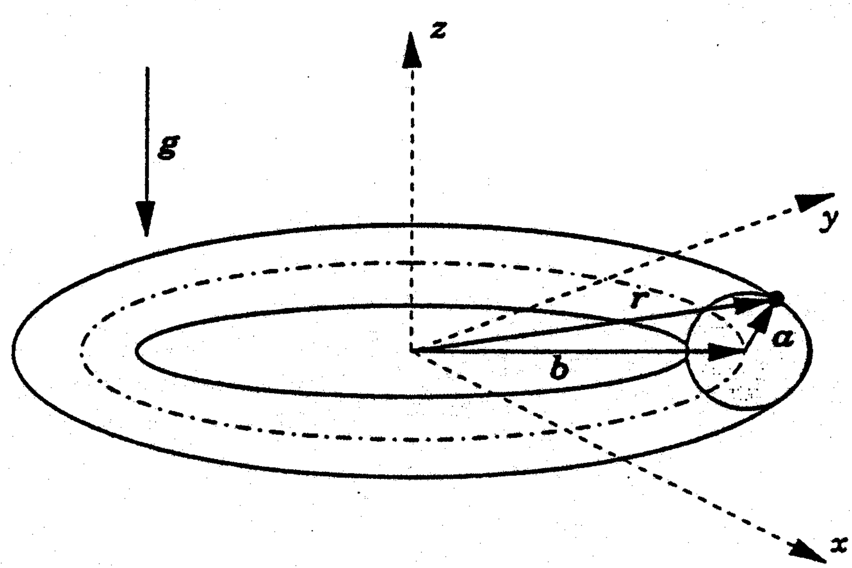
Part II

Instructions: Work all problems. This is a closed book examination. Calculators may not be used. Start each problem on a new pack of yellow paper and use only one side of each sheet. All problems carry the same weight. Write your student number on the upper right-hand corner of each answer sheet. All sheets which you receive should be handed in, even if blank.

II-1. Set up the Lagrangian of a point mass m constrained to move on a torus of major radius b and minor radius a, in the presence of a uniform downward-directed gravitational field **g** as shown. (The center of symmetry of the torus is at the origin, and the axis of symmetry of the torus is the z-axis. The minor radius is the radius of the circular cross section of the torus. The major radius is the distance from the symmetry axis to the center of the torus' cross section.) A point on the torus may be specified by $\mathbf{r}=\mathbf{a}+\mathbf{b}$ or by two generalized coordinates ϕ and θ. ϕ $(0\leq\phi<2\pi)$ is the angle between **b** (which lies in the x-y plane) and the x-axis. θ $(0\leq\theta<2\pi)$ is the angle between the positive z-axis and **a** (measured outward away from the z-axis).

a. Calculate the Lagrangian equations of motion in the two generalized coordinates.

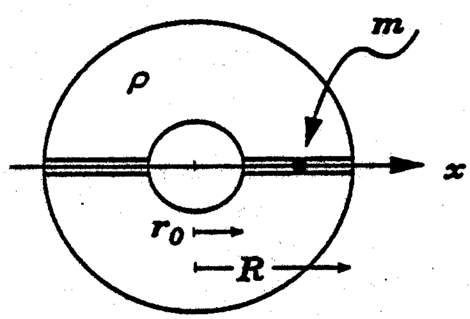
b. State which coordinate is ignorable, and thereby derive an equation of motion which involves only the other generalized coordinate and constant(s) of the motion.



II-2. A ball of uniform mass density ρ and radius R has a hollow center of radius r_0. A very thin hole is drilled through the center of the ball from left to right along the x-axis. A particle of negligible size with mass m sits initially on the x-axis at x.

a Calculate the particle's potential energy, $V(x)$, for all values of x, both inside and outside the ball.

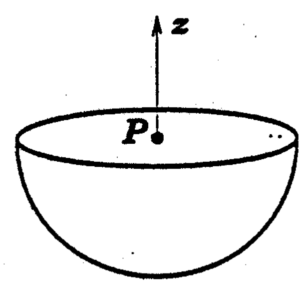
b In the limit $r_0 \rightarrow 0$, find the period of oscillation for the subsequent motion when the particle is released from rest at the point x_0, where $0<x_0<R$.



II-3. A hemisphere of radius R contains a uniform volume charge density ρ. The point P lies at the center of its circular face.

a What is the electric potential at the point P?

b What is the magnitude and direction of the electric field at the point P?



II-4. Isotropic electromagnetic radiation (*i.e.*, a superposition of plane waves arriving uniformly from all directions) falls, in vacuum, upon a perfectly absorbing surface. The energy density of the radiation is W. Find the pressure exerted by the radiation on the surface.

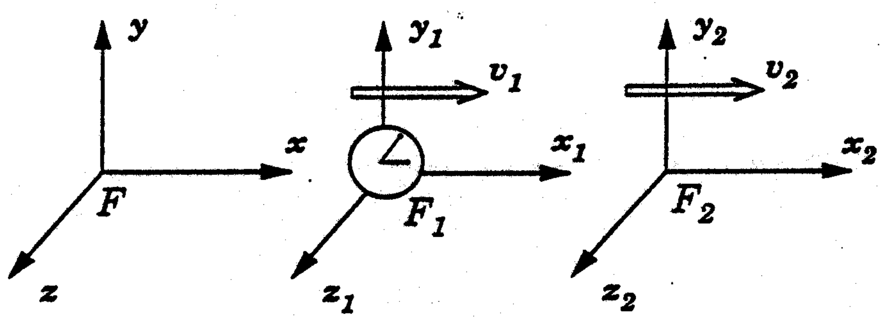
II-5. An electron with momentum p moves at a distance r from a stationary proton. If the electron is bound to the proton to form a hydrogen atom, its average position is at the proton, but the uncertainty in its position is approximately equal to the radius r of its orbit. Treat the atom as a one-dimensional system.

a Estimate the uncertainty in the electron's momentum in terms of r.

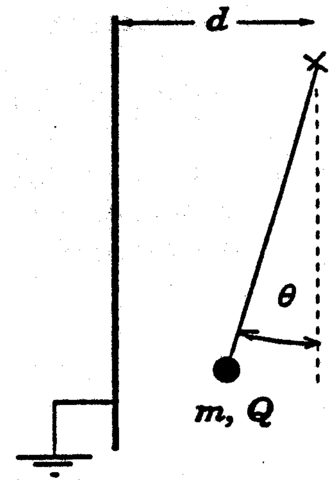
b Estimate the electron's kinetic, potential, and total energies in terms of r.

c Find the value of r which minimizes the total energy, and the corresponding value for the total energy. Compare your results with the predictions of the Bohr theory.

II-6. Two inertial reference frames F_1 and F_2 move along the x-axis of reference frame F with velocities relative to F of v_1 and v_2, respectively. A clock C is at rest in F_1. An observer in F measures the time it takes for one hand of the clock to go around once to be T. Determine the time it takes for the clock hand to go around once as measured by an observer in F_2.



II-7. A simple pendulum consists of a point mass m at the end of a massless string of length L. The pendulum is placed at a distance d from an infinite grounded conducting plane which is vertical. A positive charge Q is then placed on the point mass.



a What angle (θ_0) does the pendulum make with the vertical when in equilibrium? (You may leave your answer in the form of a transcendental equation for θ_0.)

b Find θ_0 when Q is <u>very</u> small.

II-8. A slurry (a homogeneous mixture) of oil and iron filings has a density ρ and a large magnetic susceptibility χ. One end of a vertical glass tube is submerged in a dish which contains the slurry. The tube is surrounded by a solenoid which is connected to a battery. The solenoid produces a constant magnetic intensity H within the tube. How high will the slurry rise in the tube?