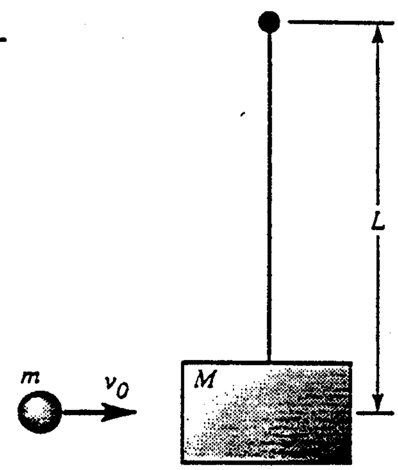
PHYSICS GRADUATE SCHOOL QUALIFYING EXAMINATION

August 16, 1995

Part I

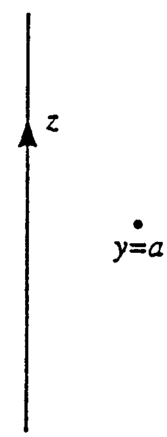
INSTRUCTIONS: Work all problems. This is a closed book examination. Start each problem on a new pack of yellow paper and use only one side of each sheet. All problems carry the same weight. *Write your student number* on the upper right-hand corner of each answer sheet.

1. A mass m with velocity v_0 makes an inelastic collision with a second mass M that is suspended by a string of length L. The velocity v_0 is perpendicular to the vertical string. After the collision the combined masses, $M+m$, rotate in a vertical plane around the point of suspension of the string. Find the minimum value for the velocity v_0 such that the string remains always under tension for a complete rotation.

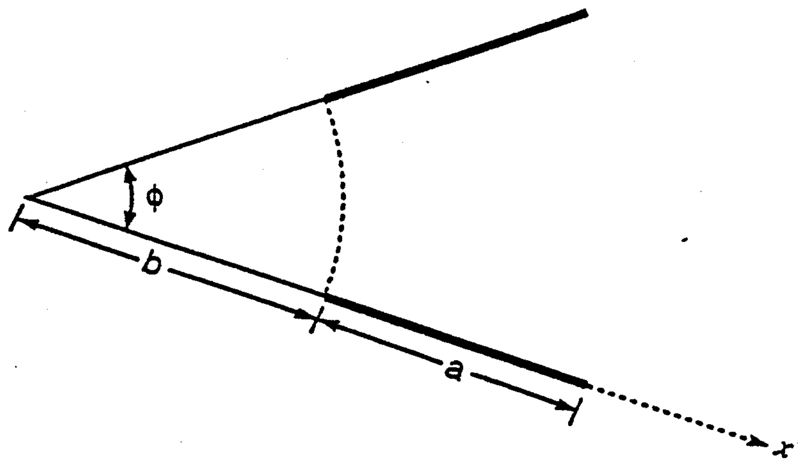


2. A person on earth can jump about one meter vertically. Estimate the radius of the largest planet with the same mean mass density as the earth from which this person can jump off completely. (The radius of the earth is about 6380 km.)

3. A horizontal line charge of linear charge density λ lies in the x-y plane parallel to the x axis at y=a, where a > 0. The x-z plane is a grounded conductor. Find the potential in the region y > 0.



4. The figure below represents a section of a capacitor whose retangular plates are inclined at an angle ϕ . The plates are of length L, perpendicular to this section. Assume that the plates are in vacuum, and that the electric field lines are arcs of circles.



Neglect fringing effects, and determine the capacitance.

5. The energy difference between the $l = 1$ and $l = 0$ rotational states of the H_2 molecule is 1.0×10^{-2}eV. Derive an expression for the distance, a, between the two protons in H_2 in terms of the mass of the proton m_p ($\hbar c$ = 197 eV-nm and $m_p = 938$ Mev/c^2).

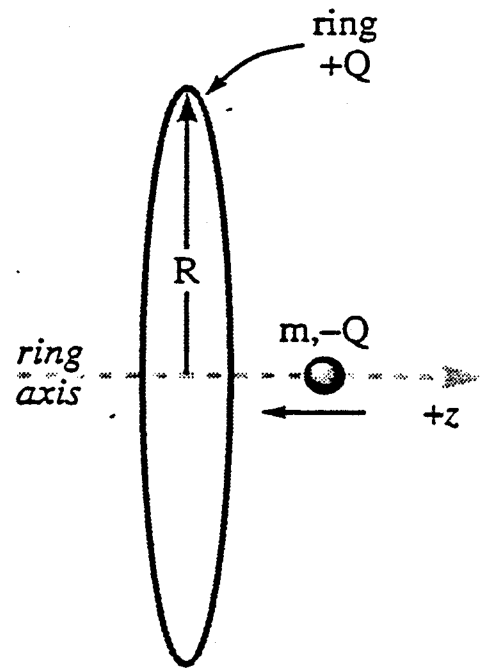
6. ^{61}Cu is a radionuclide with a half-life $t_{1/2} = 3.4$ hrs. It can be produced by a cyclotron via the nuclear reaction ^{61}Ni(d,2n) ^{61}Cu at a rate $R = 5\text{x}10^8$ atoms/sec, ^{61}Ni is stable.

Find the specific activity (# disintegrations/sec of ^{61}Cu after 1 hr, 5 hr, and 20 hr of continuous bombardment. (Assume the initial concentration of ^{61}Cu in the target is zero).

7. A particle of mass m and charge –Q is constrained to move along the axis of a thin stationary ring of radius R tha carries a uniformly distributed charge +Q.

(a) Find the frequency of the small amplitude oscillations of the particle when it is near the center of the ring.

(b) How much kinetic energy must be imparted to the particle when it is stationary at the middle of the ring for it to escape along the axis?



8. In the Hall effect, a thin slab of metal carries a current density J_x along one axis. An external magnetic field H_z is oriented normal to the face of the slab. Under steady-state, what is the magnitude of the transverse electric field E_y that is developed between opposite sides of the slab. State the answer in terms of the electron density n.

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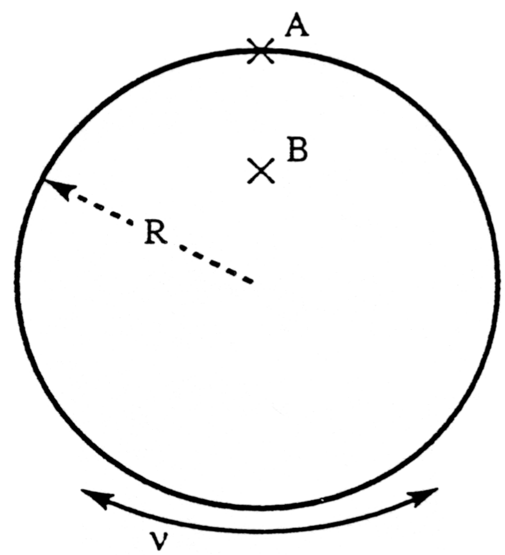
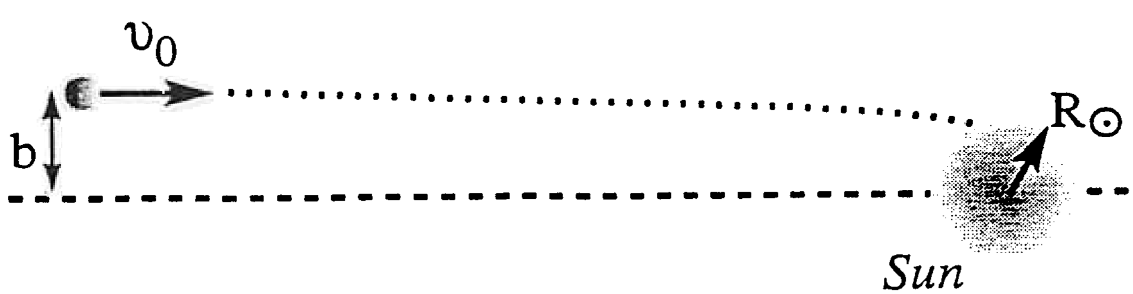
Part II

INSTRUCTIONS: Work all problems. This is a closed book examination. Start each problem on a new pack of yellow paper and use only one side of each sheet. All problems carry the same weight. *Write your student number* on the upper right-hand corner of each answer sheet.

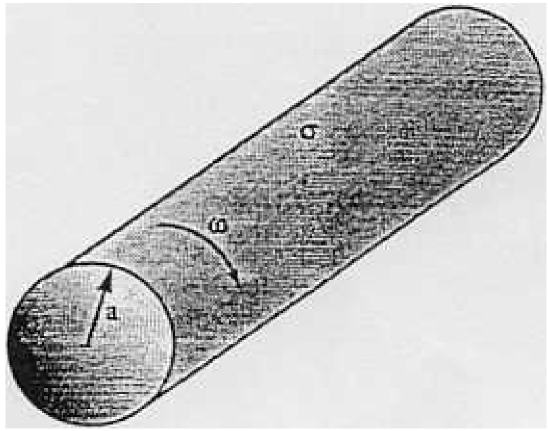
1. A disk of mass M and radius R is suspended on an axis, A, perpendicular to its circular surfaces. Axis A lies on the perimeter of the disk. The disk oscillates about that axis in small amplitude simple harmonic motion at frequency ν. There is a second axis, B, parallel to this one, for which the frequency has the same value. Find the frequency, ν, and the position of the second axis.



2. A comet approaches the sun from infinity with an initial velocity υ_0. The mass of the sun is $M_\odot$ and its radius is $R_\odot$. Find the largest impact parameter, b, for which the comet will strike the sun. Take the sun to be at rest and ignore any other bodies.



3. A long, massless, insulating cylindrical shell of radius a has a surface charge density of σ coulombs per unit area.



(a) If the cylinder rotates at an angular frequency ω about the center axis, what is the magnetic induction field B inside the cylinder?

(b) If the cylinder undergoes a constant angular acceleration so that $\omega = \dot{\omega}\, t$, what is the electric field E inside the cylindrical shell?

(c) What is the torque per unit length of the cylinder necessary to produce the angular acceleration $\dot{\omega}$? (Assume that the mass of the shell is zero.)

4. At time $t = 0$ a charge distribution $\rho(r)$ exists within an idealized homogeneous conductor whose permitivity ε and conductivity σ are constants. Obtain $\rho(r,t)$ for subsequent times.

5. (a) Derive an expression for the first (lowest energy) Bohr orbit of a negative muon (mass is 207 electron masses) around a nucleus with Z protons.

(b) Assume that a nucleus has equal numbers of protons and neutrons. Derive an expression for the value of Z for which the first Bohr orbit is equal to the nuclear radius.

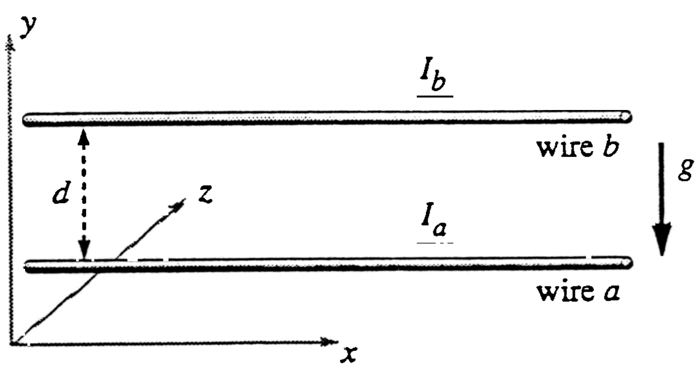
6. A W boson, a point particle of rest M_0, decays into an electron and a neutrino. If we determine that the momenta of the decay electron and neutrino are collinear and that the electron has momentum p_e in the positive x direction, what are the possible momenta of the neutrino?

7. Assume the sun has a uniform density, has a mass of $M_\odot \sim 2 \times 10^{33}$ g, and contains about 10^{57} particles.

(a) Estimate the total gravitational binding energy of the sun.

(b) Using the Virial Theorem ($\langle T\rangle = -\langle V\rangle/2$) for the $1/r$ potential and the result of (a), estimate the average internal temperature of the sun ($R_\odot = 7 \times 10^{10}$ cm).

8. A long horizontal wire a is rigidly supported in the xy plane, and carries a current I_a. Directly above it a distance d, and parallel to the wire a, is a second wire b carrying a current I_b. Wire b can move freely, but its motion is constrained to the xy plane. Wire b has mass per unit length m/L. The currents are such that wire b is supported by magnetic repulsion.



Wire b is displaced from position d by a small distance Δy and released. Show that it undergoes simple harmonic motion with the same frequency as a simple pendulum of length d. The acceleration of gravity is in the $-y$ direction and is given by the constant g.